



**enVisionMATH**  
**emBracing the Vision**  
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April 2010

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**enVisionMATH**  
**Different by Design**  
**Different to achieve certain goals relative to learning and teaching!**

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Write an equation that shows the relationship between feet and yards.  
**Let  $f$  = the number of feet**  
**Let  $y$  = the number of yards**

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**Syntax (words and their order) is emphasized rather than semantics (meanings).**

**NOT GOOD**

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**Math Makes Sense!!**



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*enVisionMATH*  
**A Research-Based Program**

**SESSION GOALS**

- **Effective Curriculum (International Studies)**
- **Effective Content Development**
  - **Develop the Concept**
  - **Solving Word Problems**

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International Research  
Effective Curriculum

- **Focus** (Not a “mile wide.”)
- **Depth** (Not an “inch deep.”)
- **Coherence** (More than a collection of activities)

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enVisionMATH - NOT a Mile Wide

“All CA nothing but the CA,  
so help me Arnold!”

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Grade-Level Organization

Focus & Depth

20 Topics Grades 1-5  
16 Topics Grade K  
Customized Sequence Possible

EACH TOPIC  
Focuses on One Strand  
A Few Related Standards

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National Council of Teachers of Mathematics (NCTM)

“A curriculum is more than a collection of activities: it must be **coherent**...”

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**Coherent Mathematics Content**

- *Skills-Development Skeleton* – CA Standards
- *Concept-Development Skeleton* – 20 **Big Ideas** & related **Essential Understandings**. (UbD)

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**Big Ideas & Essential Understandings**

**“We understand something if we see how it is related or connected to other things we know.”**

J. Hiebert, Signposts for Teaching Mathematics through Problem Solving In F. Lester & R. Charles, *Teaching Mathematics Through Problem Solving*, Grades PreK-6. NCTM: Reston, VA, 2003.

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**Research Effective Teachers**

- ✓ **Have their own mathematics content knowledge anchored on big ideas.**
- ✓ **Use big ideas as the glue for teaching, learning, and assessment (connections).**

Ma, Liping. Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum, 1999.

Stigler, James. "The Teaching Gap: Reflections on Mathematics Teaching and How to Improve It." Paper presented at the Pearson National Educational Leadership Conference, Washington, D.C., March 2004.

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Are you emphasizing

**Skills**

**AND**

**Ideas/Understandings?**

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Research-Based Program

- **Effective Content Development**
  - **Develop the Concept**

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**Research Fact:  
Effective Curriculum & Instruction**

Understanding and skill are best developed through a *balance* between student-student interactive learning and the teacher providing information at the right time and in the right way.

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**A New Vision for Basal Programs**

Site for Learning	Type of Learning Emphasized	
	Conceptual Understanding, Reasoning, Problem Solving	Skill Development
Interactive Learning Activities Outside of a Textbook	enVisionMATH	enVisionMATH
Textbook with Teacher Directed Lessons	enVisionMATH	enVisionMATH

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**enVisionMATH  
Four-Phase Instructional Model**

- 1) Daily Mixed Review
- 2) Develop the Concept: Interactive Learning
- 3) Develop the Concept: Visual Learning
- 4) Close/Assess - Diagnosis/Intervention

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**Develop the Concept: Interactive Learning**

***Problem-Based  
Interactive Learning***

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Research shows that understanding develops during the process of solving problems in which important math concepts and skills are embedded (Lester & Charles, 2003).

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**Research Fact**

“...if we want students to understand mathematics, it is more helpful to think of understanding as something that results from solving problems, rather than something we teach directly.” (Hiebert et al, 1997, p. 25)

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**All activities are NOT problem-based interactive learning experiences.**

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**Non-Problem-Based Activity**

- Totally or mostly teacher directed
- Low cognitive demand
- Too brief – no think time

**LONGER activities like this are NOT going to provide better conceptual development!**

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**Problem-Based Interactive Learning**

Phase 1: Solving and Discussing a Problem

- Teacher poses the problem,
- Students work together; teacher facilitates,
- Students share their thinking and work.

Phase 2: Making the Important Math Explicit

- Teacher-Directed Instruction
- "Classroom Conversation"

**10 to 20 minutes**

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**Problem-Based Interactive Learning Benefits**

- ✓ **Concepts and skills make sense.**
- ✓ **Concepts and skills are remembered.**
- ✓ **Concepts and skills are more effectively used in problem-solving situations.**
- ✓ **Learning gaps and misconceptions are confronted.**

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**Making the Transition to PBIL**

Goals first 3 months:

- Establish a positive environment
- Become a facilitator rather than a director
- Promote student learning independence

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**enVisionMATH**

**Develop the Concept:**  
**Problem-Based Interactive Learning**

Introduce new ideas by giving kids a chance to think (John Van de Walle).

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**Visual Learning Bridge**

**Visual Learning Animation**

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**“Word Problems”**

**Biggest “leap” in teaching problem solving since 1985!**

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**Why focus on word problems?**

- **Mathematical literacy.**
- **About 50% of the state assessment items are word problems.**

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**Why focus on word problems?**

- **Test data show that performance solving word problems is low for too many students.**
- **Teachers report that teaching word problems remains as one of their most challenging and frustrating tasks.**

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Why focus on word problems?

- **Developing students' abilities to solve word problem is critical for algebra readiness.**

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Solve

Carrie has 135 U.S. stamps. She has 3 times as many foreign stamps as U.S. stamps. How many stamps does she have all together?

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Research Fact

**Problem solving is NOT a skill.**

**Successful problem solvers are those most capable of using "quantitative reasoning."**

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### Quantitative Reasoning

**Understanding the quantities involved and how they are related, independent of the specific numbers.**

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### Research Fact

- Training children in the process of using diagrams to solve problems results in more improved problem-solving performance than training students in any other strategy.

(Yancey, Thompson, and Yancey, 1989).

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enVisionMATH

## **“Bar Diagrams”**

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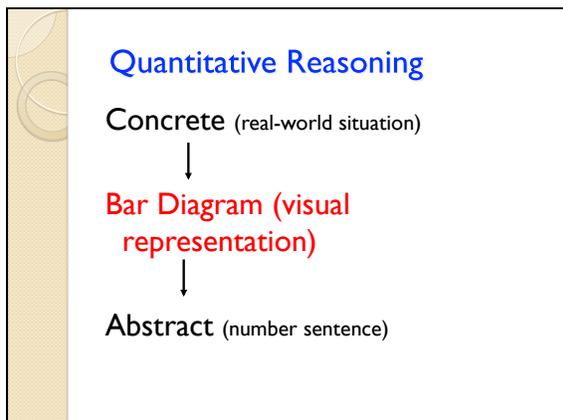
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**Quantitative Reasoning**

Carrie has 135 U.S. stamps. She has 3 times as many foreign stamps as U.S. stamps. How many stamps does she have all together?

U.S. stamps → 

135
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Foreign → 

135	135	135
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 3 times as many

$(3 \times 135) + 135$  or  
 $4 \times 135 = ?$

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**What operation(s)?**

Rick has 147 apples.  
 He ate 28.  
 How many apples does Rick still have to eat?

$147 - 28 = ?$   
 $28 + ? = 147$

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### Bar Diagrams- Joining Equal Groups

Tom has 4 boxes of projector pens. There are 5 pens in each box. How many in all?

Joining Equal Groups: Total Amount Unknown

$4 \times 5 = ?$

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### Bar Diagrams- Joining Equal Groups

Pam had 5 bags and put the same number of apples in each bag. She ended up with 45 apples in bags. How many did she put in each bag?

$5 \times ? = 45$   
 $45 \div 5 = ?$

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### Bar Diagrams- Separating Equal Groups

Byron has 45 pigeons. He keeps them in 5 pens with the same number of pigeons in each. How many pigeons are in each pen?

$45 \div 5 = ?$

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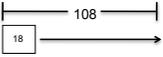
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**Bar Diagrams- Separating Equal Groups**



A total of 108 children signed up for soccer. How many 18-person teams can be made?

$108 \div 18 = ?$

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**A Developmentally Powerful Sequence from Grade K-6**

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**Bar Diagrams**

**Focus I: Number meanings & relationships**

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**Bar Diagrams**

**Focus 2: Operation meanings & relationships**

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**Bar Diagrams**

**Focus 3: Representing & solving word problems**

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**Success Starts with YOU**

**We must believe that ALL students can understand mathematics, develop computational fluency, and become successful problem solvers.**

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