Lead-In Materials Overview: Integrated Algebra

The performance assessment test you will be administering to your Algebra 1 classes is aligned to the Common Core State Standards in Mathematics. These tasks were specifically developed to assess key Common Core standards in High School Algebra. This assessment is comprised of five tasks. Each of the tasks addresses several standards. These tasks are designed to assess a learning progression of knowledge and understanding of essential concepts and skills in a math course. The tasks probe deeper to measure students' conceptual understanding and how they can apply, generalize and/or justify their findings. The purpose of these tasks is to allow students to fully show what they know and are able to do using key concepts in a math course.

Overall Assessment Structure	
Session 1: Introduction	Familiarize students with a medium scaffolded performance task. This will provide a learning experience where students can understand the format and expectations of the assessment instrument.
Session 2: Day 1 of Task administration	Administer the **first three** tasks to the student.
Session 3: Day 2 of Task Administration	Administer the **last two** tasks to the student.

Teacher Lead-in Materials **Session 1**

(25 min.) **Activity 1**: Administer Sample Task

1) Pass out student lead-in booklet which contains a sample task
2) Explain to the students that the purpose of assessment is to see what they know, or what they need to learn. Emphasize that they should attempt to do the entire task and that if they feel they can not, they should work on as much of it as possible. Ensure that students understand that their answers can help them know what they need to focus on learning this year.
3) Allow students to work with the task individually.
    a. Students should show and explain all their work and thinking. The more students show their work and explain their thinking, the better they will score on the tasks.
    b. Encourage students to never erase, rather to draw a line through the erroneous part of their answers.

   c. Encourage students to create diagrams, pictorial representations, tables, graphs, number sentences and equations.
   d. Students should do their best to reflect and recall their thinking, and then share that as part of the written response.
4) Allow all students to finish the task.

(20 min.) **Activity 2**: View the rubric and samples of student work.

1) Show students the rubric for the task.
2) Use the samples of student work to show students the range of solution methods and strategies.
3) When reviewing the task, share interesting solutions.
   a. Help students understand the expectations of performance.
      i. Often, there are different approaches or different solution paths that are worthy of credit.
      ii. Help students realize that different methods are often awarded the same points even though the approaches vary and maybe less mathematically sophisticated than others.
   b. Students should understand that giving an honest attempt and showing all of their thinking is beneficial.
   c. If possible (time permitting) help students understand how even answering questions wrong can help them understand what they know and what they need to learn.
4) If you have time: Review the Questions for Reflection on Picking Apples.

Picking Apples

This problem gives you the chance to:
• work out costs from given rules

Anna goes to pick apples.
She sees two orchards next to each other; David's orchard and Pam's orchard.
The signs below are at the entrance to the orchards.

DAVID'S APPLE ORCHARD Pick your own apples!	PAM'S ORCHARD DELICIOUS APPLES
First 10 pounds $2 per pound	$10 entry fee
Each additional pound $1 per pound	First 10 pounds $1.50 per pound
	Each additional pound $0.75

Anna wants to pick 40 pounds of apples.

1. a. How much does this cost at David's orchard? _____

   Show your calculations.

   b. How much does it cost at Pam's orchard? _____

   Show your calculations.

Chris has $30 to spend.

2. a. How many pounds of apples will he get if he goes to David's orchard? _____

Explain how you figured it out.

_____

_____

_____

b. If Chris goes to Pam's orchard, how many pounds of apples will he get? _____

Explain how you figured it out.

_____

_____

_____

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

Show your work. _____

Picking Apples	Rubric	
The core elements of performance required by this task are: • work out costs from given rules Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. a. Gives correct answer: **$50**	1	
Shows correct work such as: 10 x $2 + 30 x $1	1	
b. Gives correct answer: **$47.50**	1	
Shows correct work such as: $10 + 10 x $1.50 + 30 x $0.75	1	4
2. a. Gives correct answer: **20 pounds**	1	
Gives a correct explanation such as: The first 10 pounds of apples cost $20. The remaining $10 buys 10 pounds. Altogether 10 + 10 = 20 pounds.	1	
b. Gives correct answer: **$16\frac{2}{3}$ pounds** (accept 16)	1	
Gives a correct explanation such as: The entry fee is $10. The first 10 pounds of apples cost $15. The remaining $5 buys 6.6 (accept 6) pounds. Altogether 10 + 6.6 = 16.6 pounds (accept 16)	1	4
3. Gives correct answer: **more than 30 pounds** (Accept 31)	1	
Shows work such as: David's: 10 x $2 + 20 x $1 = $40 Pam's: $10 + 10 x $1.50 + 20 x $0.75 = $40 **or** Draws a correct graph	1 or 1	2
Total Points		**10**

Looking at Student Work on Picking Apples

Student A shows a clear understanding of the proportional relations described in "per pound" by showing the multiplication for the different amounts of apples. The student uses labels clearly to define what each computation represents. To solve for part 3 the student makes an organized list to show where David's cost is less than Pam's, at what point the costs are the same, and the where Pam's becomes less expensive.

Student A

Anna goes to pick apples.
She sees two orchards next to each other; David's orchard and Pam's orchard.
The signs below are at the entrance to the orchards.

DAVID'S APPLE ORCHARD Pick your own apples!	PAM'S ORCHARD DELICIOUS APPLES
First 10 pounds $2 per pound	$10 entry fee First 10 pounds $1.50 per pound
Each additional pound $1 per pound	Each additional pound $0.75

Anna wants to pick 40 pounds of apples.

1. a. How much does this cost at David's orchard? $50 (4) ✓✓ 1

Show your calculations.

10 × 2 = $20 for first 10 lbs.

40 − 10 = 30 lbs. left

30 lbs × $1 per lb. = $30

$30 + $20 = $50 ✓ 1

b. How much does it cost at Pam's orchard? $47.50 ✓✓ 1

Show your calculations.

$1.50 × 10 = $15 for first 10 lbs.

30 lbs. left × .75 per each additional lb. = $22.50 ✓✓ 1

$15 + $22.50 = $37.50

+ $10.00 entrance fee

$47.50

Student A, part 2

2. a. **How many pounds of apples will he get if he goes to David's orchard?** 20 pounds ✓ 1
Explain how you figured it out.

The first 10 pounds is $20, because 10lbs × $2 = $20. Chris then only has $10 left to spend, and each additional pound after buying the first 10lbs. is $1. So he can buy 10lbs of apples.

b. **If Chris goes to Pam's orchard, how many pounds of apples will he get?** 116 pounds ✓✓ 1
Explain how you figured it out.

$10 of Chris' money is spent on the entrance fee. So, he only has $20 to spend on apple at Pam's orchard. The first 10lbs. at Pam's costs him $15. He has $5 left to spend, but apples are 75¢ now that he has bought his first 10lbs. If he buys 6 more pounds he will spend $4.50. (6×.75=4.50) And he will spend a total of $29.50

3. **How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?**
Show your work. 31 pounds ✓ 1

Davids

lbs.	$
19	29
20	30
21	31
25	35
30	40
31	41

Pams

lbs.	$
19	31.75
20	32.75
21	38.19
25	36.25
30	40
31	40.75

19 31.75

1
10
15
5
40

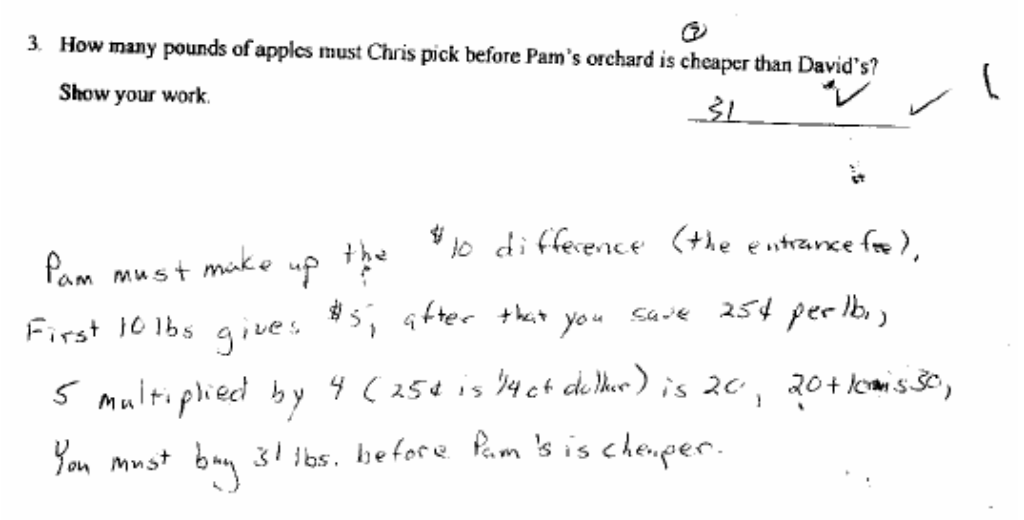
10.00
15.00
15.75

10
15
7.50

✓ 1

Student C approaches the problem from a different perspective by identifying the cause for Pam's initial higher costs, i.e. the entrance fee. The student then shows how many pounds must be purchased to compensate for that initial cost.

Student C



3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

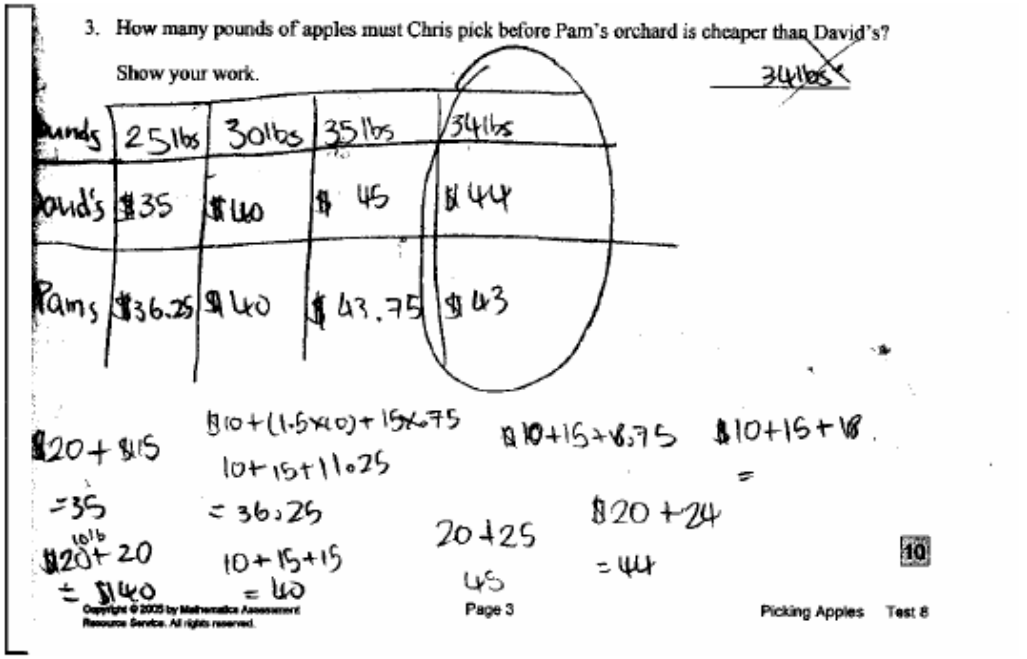
Show your work.

$1

Pam must make up the $10 difference (the entrance fee).
First 10 lbs gives $5, after that you save 25¢ per lb,
5 multiplied by 4 (25¢ is ¼ of dollar) is 20, 20+10 is 30,
You must buy 31 lbs. before Pam's is cheaper.

Student D tries to use the organized list to find where Pam's cost is less than David's. The student knows that the change occurs between 30 lb. and 35 lb. The student's logic breaks down by forgetting that he is looking for the "smallest" amount where Pam's is cheaper rather than any amount where Pam's is cheaper. When looking at the table of values for 30 lb. and 35 lb., the student should have jumped to 31lb.

Student D



3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

Show your work.

34lbs

unds	25lbs	30lbs	35lbs	34lbs
vid's	$35	$40	$45	$44
Pams	$36.25	$40	$43.75	$43

$20+$15 = 35

$10+(1.5×10)+15×.75
10+15+11.25
= 36.25

$10+15+8.75

$10+15+18 =

$20+20 = $40 (10lb)

10+15+15 = 40

20+25 = 45

$20+24 = 44

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Picking Apples

Points	Understandings	Misunderstandings
0	92% of the students with this score attempted the task.	Some students had difficulty interpreting the language of proportionality, "per pound". 8% of all students had answers of $32 for apples at David's, because they didn't multiply the first 10 lbs. by $2. 8% of the students thought the cost of 40 lbs. was $40, just taking $1 for every pound and ignoring the difference in price for the first 10 lbs.
2	Students could interpret the meaning of "per pound" and calculate the cost of 40 lbs. of apples at David's orchard.	Students had difficulty interpreting the 3 constraints for Pam's orchard. 11% of the students ignored the entry fee. 4% only added $1.50 for the first 10 lbs. getting answer of $34. 5% did not use monetary notation giving an answer of $47.5.
4	Students could calculate the cost of buying 40 lbs. of apples at David's and find the number of pounds that could be purchased for $30, showing appropriate calculations for each.	11% of the students, who missed part 2a, ignored the change in cost for the first 10 lb. They thought $30 would get 30 lbs. 6% thought that if 40 lbs. = $40 and the first 10 lb. = $2, then $30 would buy 38 lbs. Other common answers for 2a were 15, 150, and 40 pounds.
6	Students could find the cost of 40 lbs. at both orchards. They could work backwards from a cost of $30 to the amount of apples purchased at David's, but not at Pam's.	Some students had difficulty working backwards from a cost of $30 to the number of pounds at Pam's. Some calculated it as if all apples cost $ .75, giving them an answer of 40 lb. Some students did not realize you can by fractional amounts of pounds, so they picked answers that would use most of the money like 13 or 15 pounds.
8	Students could find the cost of 40 lbs. or the number of pounds that could be purchased for $30 at both orchards, showing appropriate calculations.	Many students who were successful at all other parts of the task did not attempt the final part of the task. They did not know how to attack the problem. 65% of all students did not attempt this part of the task.
10	Students could find the cost of 40 lbs. or the number of pounds that could be purchased for $30 at both orchards, showing appropriate calculations. Students could also find the point where Pam's orchards were cheaper than David's.	Students might try guess and check to find some value where Pam's orchard was cheaper, but not narrow it down to the lowest amount giving answers as high as 100 or 187 lbs.

Questions for Reflection on Picking Apples

- What kinds of language do your students use to make sense of rates or proportional situations? Do you think they understand terms like per pound, per hour, per box? Do you think they see these terms as sets of equal size groups?
- What types of activities do students do to help them make sense of the meaning behind rates or proportions? Do they associate multiplication/ division with these ideas? In what ways?
- How are labels used in the classroom when solving problems? Do you provide explicit instruction to help students deal with dimensional analysis or how operations effect or change labels?
- Looking at student work in part 1 and 2, were students thinking in terms of function or doing several individual calculations?

Look at student work for part 1a. How many of your students put:

$50	$40	$32	$8	$30	$20	Other

Can you follow the reasoning chain that led to each particular error pattern? What does this show you about student misunderstandings?

Look at student work for 1b. How many of your students put:

$47.50	$47.5 or $47.05	$55	$34	$37.50	Other

What additional misconceptions contributed to the problems in this part of the task?

- What types of problems do students work requiring them to "work backwards" or do inverse operations? Do students work with this idea with computational procedures or just with problem-solving tasks?
- Are students comfortable with order of operations and how that works when undoing a procedure?

- How comfortable are students with symbolic notation like parentheses, division symbols?
- When looking at student work in part 2, check student thinking to see if they combined inappropriate terms like $ and pounds. Did they lose track of the meaning behind their computations?

Look at student work on 2a. How many students put:

20	15	30	150	38	40	Other

Look at student work on 2b. How many students put:

16 2/3 or 16	10	30	19	20	25	No response	Other

What misconceptions led to these error patterns? What made this part more difficult for students?

Now look at work in the final section. Many students did not know how to approach this part of the task. They did not have the sense of finding the point where the number of pounds and the cost were the same for both orchards.

- How many of your students did not attempt this part of the task?
- How many of your students guessed a large value (34 to 187) that made Pam's cheaper?
- How many lost track of some of the constraints (like entrance fee) when making their calculations for this section?
- Were successful students able to set up an equation to solve the problem or did they use guess and check to solve the problem?

Do you have any problems in your text dealing with the idea of break-even point? Have you students worked with problems graphing two equations to find where they intersect? What strategies would you have expected or wanted your students to be able to use?

Implications for Instruction:

Students at this grade level should be comfortable with identifying and using constraints to solve problems. Students should be starting to do operations with labels to keep track of how the calculations change the labels. Students should also start to use equations to express the multiple constraints, rather than using a string of calculations.

A big idea for middle grades is the ability to use proportional reasoning or understand multiplicative relationships. Students should be comfortable with the language of proportions or rates, like per pound, per hour, per person. Students need explicit instruction of help them connect cost per pound or miles per hour as representing equal groups that can be multiplied or divided. Students also need help seeing how these operations change the units.

Students at this grade level should be preparing for the transition to algebra. They need many opportunities to work problems involving inverse operations. They need to be confronted with situations with multiple steps, where order of operations makes a difference. So when students share solutions, it is important for them to be asked why they have different answers. The teacher might pose questions, such as, "If two students both subtracted and both divided, how is it possible for them to get two different solutions? How can we determine which one is correct?" Having students grapple with these issues helps them see the logic behind the rules or algorithms in a way that direct instruction alone doesn't.

Another big idea to help then prepare for algebraic thinking is the idea of equality. In order for students to think about the idea of when Pam's orchard is cheaper, it is helpful to ask the question, " At what point are the two orchard's the same?" As more than half the students had no idea how to even start this part of the task, giving them this as a discussion point when returning the papers would be a good classroom activity.
